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**MATH201 - Calculus-I**

**Homework Assignment #4**

**Due day: 8/5/2023**

1. (a) Graph in Excel the function in the viewing rectangle [-2π, 2π] by [-4, 4]. What slope does the graph appear to have at the origin?

ANS: It appears as a tangent line. The slope is close to 0

F’(x) = =0

F’(x) = cos(x) - × 1000 cos (1000x) =0

Cos(x) = cos(1000x)

Cos(x) = cos (x+2 π)

1000x = x+2 π

999x=2 π

X=  ͌ 0.00628

Or

1000x= -x+2 π

1001x=2 π

X= ͌ 0.00628

F’(x) = = cos(x) - × 1000 cos (1000x)

= cos(x)-cos(1000x)

When x=0 f(x) ͌ 0 The slope is close to 0

(b) Zoom in to the viewing window [-0.4, 0.4] by [-0.25, 0.25] in Excel and estimate the value of . Does this agree with your answer from part (a)?

ANS: This agrees with the answer from part a.

F’(0) = = = 0

(c) Now zoom in to the viewing window [-0.008, 0.008] by [-0.005, 0.005] in Excel. Do you wish to revise your estimate for ?

ANS: As x close to 0, it is not a tangent line.

1. Graph in Excel the function . Zoom in repeatedly, first toward the point (-1, 0) and then toward the origin. What is different about the behavior of *f* in the vicinity of these two points? What do you conclude about the differentiability of *f* ?

ANS: From the point (-1,0) toward the origin, the y value increase as x increase, as x getting closer to 0, y value decrease.

1. The local maximum from the point of -1 to 0,

X’+ ’ = 0

1+ =0

1=

= -2

= -

Therefore f’(x) does not exist.

The graph show the minimum value take on more than once at [-1, 0], and [0.0]

1. The behavior at point (-1,0), is continuous and differentiable.

= =  = = = = =0

1. At the point of (0,0), tangent line is horizontal, is not differentiable.
2. The left-hand and right-hand derivatives of *f* at *a* are defined by

and

if these limits exist. Then exists if and only if these one-sided derivatives exist and are equal.

1. Find  and for the function

ANS:

when = 0 = 0

when = 5-x = (5 - x) *= 0-1=-1*

*when*  *=*

*= = = = = =*

(b) Sketch the graph of *f*.

(c) Where is *f* discontinuous?

F discontinuous at x=0 and x= 5

(d) Where is *f* not differentiable?

When , *f not differentiable since f’(x) does not exit.*

1. If *f* is a differentiable function and , use the definition of a derivative to show that

ANS:

If f is a differentialbe function, f’(x) exit.

g’(x)= [x f(x)]’ = x’ f(x)+ f’(x) x = 1 f(x) +x f’(x) =

g’(x)=

since g(x) =x f(x)

g’(x) = =  = = -

= -

Since f’(x) = = and f(x) = f(x+h)

Therefore,

1. Boyle’s Law states that when a sample of gas is compressed at a constant temperature, the pressure *P* of the gas is inversely proportional to the volume *V* of the gas.
   1. Suppose that the pressure of a sample of air that occupies *0.106* at *25℃* is *50 kPa*. Write *V* as a function of *P*.

ANS: V= *0.106* T= *25℃ (a constant)* *p= 50 kPa*

Since V =

Therefore c = V P = 0.106 ×50 = 5.3

V=

* 1. Calculate when *P = 50 kPa*. What is the meaning of the derivative? What are its units?

ANS: If *P1= 50 kPa* V1= *0.106*

*P2 = 60kPa V2= = = 0.088*

*= = = = 0.0018*

*= = f’(p)*

*The units are the rate of change of volume caused by the change of pressure. /kPa.*

1. Car tires need to be inflated properly because overinflation or underinflation can cause premature tread wear. The data in the table show tire life *L* (in thousands of miles) for a certain type of tire at various pressures *P* (in ).

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| *P* | *26* | *28* | *31* | *35* | *38* | *42* | *45* |
| *L* | *50* | *66* | *78* | *81* | *74* | *70* | *59* |

* 1. Use a calculator to model tire life with a quadratic function of the pressure.

Y=ax2 + bx+c x=

*= = = 8 therefore b=8*

*= = = 4*

*= = = 3*

*= = = -3.5*

*= = = -1*

*= = = -3.67*

*F’(x)= (*ax2 + bx+c )’ =0 2ax+b+0 = 0 2ax+b=0 2ax=-b x =

*F’’(x) =*  =0 =

Plot the data and form the equation as: L= -0.2754P2+19.749P-273.55

* 1. Use the model to estimate when *P = 30* and when *p = 40*. What is the meaning of the derivative? What are the units? What is the significance of the signs of the derivatives?

ANS:

F’(L) = (-0.2754P2+19.749P-273.55)’ = 2× (-0.2754 P) + 19.749-0

= ( -0.5508) P +19.749

When P=30

L= -0.2754P2+19.749P-273.55 = (-0.2754) ×302+19.749×30 -273.55

=71.06

F’(30) = (-0.5508)×30 +19.749 = -16.6524+19.749=3.225

When P=40

L= -0.2754P2+19.749P-273.55 = (-0.2754) ×402+19.749×40 -273.55

=75.77

F’(40) = = (-0.5508)×40 +19.749 = -2,283

1. The deribvative means the rate of change when the change of the tire Life caused by the change of the the tire Pressure. When tire life increases, the tire pressure decreases.
2. The units are every thousand miles of changes per .
3. The maximun of local point is [35,81]

F’(P) = 0 = ( -0.2754P2+19.749P-273.55)’ = (-2

(-2)

P= 35.855

L = -0.2754 (35.8552)+19.749×35.855-273.55 = 80.5